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POLARIZATION MATRICES OF QUARTZ

ARTHUR BALLATO AND THEODORE LUKASZEK
ELECTRONICS TECHNOLOGY AND DEVICES LABORATORY

JULY 1989

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In analytical treatments of piezoelectric-acoustic transducers, signal processors, and resonators, the electromechanical transduction mechanism is most often expressed in terms of the elements of the piezoelectric [e] or [d] matrices. Molecular interpretations of piezoelectricity, and especially electrooptical applications, usually involve polarization as the preferred variable, and consequently the alternative [a] and [b] matrices are of interest. The elements of these latter sets are calculated for quartz from measured elastopiezodielectic constants taken from the literature.

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INTRODUCTION

Electromechanical transduction taking place via the piezoelectric effect is characterized phenomenologically by constitutive equations that relate the elastic and electric variables. These equations take a variety of forms, depending upon the choice of independent and dependent variables; the choice is normally dictated by the application. For example, piezoelectric resonators in the form of thickness mode plates are most easily treated using the isagric elastic stiffnesses [cE], the piezoelectric stress constants [e], and the dielectric permittivities at constant strain [(eps)S].

Various measurement techniques yield values for the elements of a particular coefficient set more directly than those of another. The coefficients appearing in the different equation sets are, however, interrelated, so that once any one complete set is available, all the other sets of elements may be found. The most accurate and precise experimental results to date have been from plate resonator (resonance) and pulse-echo (transit-time) measurements. From the [cE], [e], and [(eps)S] matrices determined therefrom, those matrices representing material properties expressed in the other alternative forms may be calculated.

Electrooptical applications are becoming increasingly important. So also are treatments of piezoelectric and ferroelectric phenomena from the standpoint of molecular interactions. In both of these cases the constitutive equations using polarization as the independent electrical variable, rather than either electric intensity or displacement, assume greater importance than the sets traditionally used for transducer, signal processing, and resonator applications.

In this report we give the complete sets of linear constitutive equations relating elastic and electric fields. For each equation set the numerical values are computed for quartz, from the measured [cE], [e], and [(eps)S] values of Bechmann (Ref. 1). Coupling to the thermal field is neglected. Rationalized mks units are used throughout.

CONSTITUTIVE EQUATION SETS

Symbols and units for the quantities employed are given in Table 1. In terms of these, six constitutive equation sets are used. Of these, electric intensity, dielectric displacement, and polarization each appear in two sets as an independent variable. The sets are, in compressed matrix notation, as follows. A prime denotes transpose; [I] is the unit matrix.

I. The Piezoelectric Stress Constant Set

$$[T] = [cE]\,[S] - [e]'\,[E] \qquad (1)$$
$$[D] = [e]\,[S] + [(eps)S]\,[E] \qquad (2)$$

TABLE 1. SYMBOLS, UNITS, AND DEFINITIONS.

QUANTITY	UNIT	SYMBOL/DEFINITION
Elastic stress	N/m2	[T]
Elastic strain	-----	[S]
Electric intensity	V/m	[E]
Dielectric displacement	C/m2	[D]
Dielectric polarization	C/m2	[P]
Elastic compliance at constant [E], [D], [P]	m2/N	[sE], [sD], [sP]
Elastic stiffness at constant [E], [D], [P]	N/m2	[cE], [cD], [cP]
Dielectric permittivity at constant [T], [S]	F/m	[(eps)T], [(eps)S]
Dielectric constant, relative, at constant [T], [S]	-----	[(Kr)T], [(Kr)S] =[(eps)T]/(eps)o, [(eps)S]/(eps)o
Dielectric impermeability at constant [T], [S]	m/F	[(bet)T], [(bet)S] =[(eps)T] $^{(-1)}$, [(eps)S] $^{(-1)}$
Dielectric impermeability, relative, at constant [T], [S]	-----	[(betr)T], [(betr)S] =[(bet)T]*(eps)o, [(bet)S]*(eps)o =[(Kr)T] $^{(-1)}$, [(Kr)S] $^{(-1)}$
Dielectric susceptibility at constant [T], [S]	F/m	[(chi)T], [(chi)S] =[(Kr)T-I]*(eps)o, [(Kr)S-I]*(eps)o
Dielectric susceptibility, relative, at constant [T], [S]	-----	[(chir)T], [(chir)S] =[(chi)T]/(eps)o, [(chi)S]/(eps)o
Reciprocal dielectric susceptibility at constant [T], [S]	m/F	[(zet)T], [(zet)S] =[(chi)T] $^{(-1)}$, [(chi)S] $^{(-1)}$
Reciprocal dielectric susceptibility, relative, at constant [T], [S]	-----	[(zetr)T], [(zetr)S] =[(zet)T]*(eps)o, [(zet)S]*(eps)o
Piezoelectric stress constant	C/m2	[e]

TABLE 1. SYMBOLS, UNITS, AND DEFINITIONS. (continued)

QUANTITY	UNIT	SYMBOL/DEFINITION
Piezoelectric strain coefficient	m/V = C/N	[d]
Piezoelectric stress modulus	N/C = V/m	[h]
Piezoelectric strain constant	m2/C	[g]
Piezoelectric polarization modulus	V/m = N/C	[a]
Piezoelectric polarization constant	m2/C	[b]

Note: Square brackets, sic: [ ], denote matrices.

II. The Piezoelectric Strain Coefficient Set

$$[S] = [sE]\,[T] + [d]'\,[E] \qquad (3)$$
$$[D] = [d]\,[T] + [(eps)T]\,[E] \qquad (4)$$

III. The Piezoelectric Stress Modulus Set

$$[T] = [cD]\,[S] - [h]'\,[D] \qquad (5)$$
$$[E] = -[h]\,[S] + [(bet)S]\,[D] \qquad (6)$$

IV. The Piezoelectric Strain Constant Set

$$[S] = [sD]\,[T] + [g]'\,[D] \qquad (7)$$
$$[E] = -[g]\,[T] + [(bet)T]\,[D] \qquad (8)$$

V. The Piezoelectric Polarization Modulus Set

$$[T] = [cP]\,[S] - [a]'\,[P] \qquad (9)$$
$$[E] = -[a]\,[S] + [(zet)S]\,[P] \qquad (10)$$

VI. The Piezoelectric Polarization Constant Set

$$[S] = [sP]\,[T] + [b]'\,[P] \qquad (11)$$
$$[E] = -[b]\,[T] + [(zet)T]\,[P] \qquad (12)$$

The electric variables are connected by the relation

$$[D] = (eps)o * [E] + [P] \qquad (13)$$

where (eps)o is the permittivity of free space, defined by

$$(eps)o * (mu)o * (c) * (c) = 1 ; \qquad (14)$$

(mu)o is the permeability of free space, equal, by definition, to 4 PI * $10^{(-7)}$, and (c) is the velocity of light in vacuo and, also by definition, is equal exactly to 2.99792458 x 10^8 m/s.

From (13) the expressions for the remaining electric variables associated, respectively, with the six equation sets (1) to (12) may be found:

$$[P] = [e]\,[S] + [(chi)S]\,[E] \qquad (15)$$

$$[P] = [d]\,[T] + [(chi)T]\,[E] \qquad (16)$$

$$[P] = (eps)o * [h]\,[S] + [I - (eps)o * (bet)S]\,[D] \qquad (17)$$

$$[P] = (eps)o * [g]\,[T] + [I - (eps)o * (bet)T]\,[D] \qquad (18)$$

[D] = -(eps)o * [a] [S] + [I + (eps)o * (zet)S] [P] (19)

[D] = -(eps)o * [b] [T] + [I + (eps)o * (zet)T] [P] (20)

RELATIONS AMONG MATERIAL CONSTANTS

The material constants are interrelated by the following formulas:

[cX] [sX] = [(eps)Y] [(bet)Y] = [I] (21)

[(chi)Y] [(zet)Y] = [(Kr)Y - (chir)Y] = [I] (22)

In (21) and (22), X = E, D, or P and Y = T or S.

[cD] - [cE] = [h]' [e] = [e]' [h]

= [h]' [(eps)S] [h] = [e]' [(bet)S] [e]

= [a]' [e - h * (eps)o] = [e - h * (eps)o]' [a] (23)

[cP] - [cD] = [h]' [a] * (eps)o = [a]' [h] * (eps)o

= [h]' [(eps)S] [(zet)S] [h] * (eps)o

= [a]' [(bet)S] [(chi)S] [a] * (eps)o

= [a - h]' [e] = [e]' [a - h] (24)

[cP] - [cE] = [a]' [e] = [e]' [a]

= [a]' [(chi)S] [a] = [e]' [(zet)S] [e]

= [h]' [e + a * (eps)o] = [e + a * (eps)o]' [h] (25)

[sE] - [sD] = [d]' [g] = [g]' [d]

= [d]' [(bet)t] [d] = [g]' [(eps)t] [g]

= [b]' [d - g * (eps)o] = [d - g * (eps)o]' [b] (26)

[sD] - [sP] = [b]' [g] * (eps)o = [g]' [b] * (eps)o

= [g]' [(eps)T] [(zet)T] [g] * (eps)o

= [b]' [(bet)T] [(chi)T] [b] * (eps)o

= [b - g]' [d] = [d]' [b - g] (27)

[sE] - [sP] = [b]' [d] = [d]' [b]

= [b]' [(chi)T] [b] = [d]' [(zet)T] [d]

= [g]' [d + b * (eps)o] = [d + b * (eps)o]' [g] (28)

[(zet)S] - [(zet)T] = [b] [a]' = [a] [b]'

= [b] [cP] [b]' = [a] [sP] [a]' (29)

[(chi)T] - [(chi)S] = [(eps)T] - [(eps)S]

= [e] [d]' = [d] [e]'

= [d] [cE] [d]' = [e] [sE] [e]' (30)

[(bet)S] - [(bet)T] = [h] [g]' = [g] [h]'

= [g] [cD] [g]' = [h] [sD] [h]' (31)

[e] = [d] [cE] = [(eps)S] [h] = [(chi)S] [a] (32)

[d] = [e] [sE] = [(eps)T] [g] = [(chi)T] [b] (33)

[h] = [g] [cD] = [(bet)S] [e] = [(chi)S] [(bet)S] [a]

= [I - (bet)S * (eps)o] [a] (34)

[g] = [h] [sD] = [(bet)T] [d] = [(chi)T] [(bet)T] [b]

= [I - (bet)T * (eps)o] [b] (35)

[a] = [b] [cP] = [(zet)S] [e] = [(eps)S] [(zet)S] [h]

= [I + (zet)S * (eps)o] [h] (36)

[b] = [a] [sP] = [(zet)T] [d] = [(eps)T] [(zet)T] [g]

= [I + (zet)T * (eps)o] [g] (37)

Some alternative relations are the following:

[a - h] = [(zet)S] [h] * (eps)o

= [(bet)S] [a] * (eps)o (38)

[b - g] = [(zet)T] [g] * (eps)o

= [(bet)T] [b] * (eps)o (39)

[e + a * (eps)o] = [(eps)S] [a] (40)

[d + b * (eps)o] = [(eps)T] [b] (41)

[e - h * (eps)o] = [(chi)S] [h] (42)

[d - g * (eps)o] = [(chi)T] [g] (43)

Equations (21) to (43) result from equating like dependent variables in pairs selected from equations (1) to (12) and (15) to (20). Each

pair yields one equation in three variables, one mechanical and two electrical, or vice versa. Two other equations exist, again from (1) to (12) and (15) to (20), that contain the same three variables found in each paired equation. One of these auxiliary equations is used to eliminate one of the two variables of the same kind; the result is one equation in two variables, one electrical and one mechanical. These are now independent variables, so the coefficients must vanish; two relations between the material coefficients result. As an example, (3) and (7) both have [S] as dependent variable. Equating them produces one relation in [T], [E], and [D]; one of the electrical variables must be eliminated. This is done by using either (4) or (8); each contains the same three variables. If (8) is used to eliminate [E], one obtains [sE - d' g - sD] [T] = [d' (bet)T - g'] [D]. Therefore, [sE] - [sD] = [d]' [g] and [g] = [(bet)T] [d]. Use of (4) instead of (8) leads to the equations [sE] - [sD] = [g]' [d] and [d] = [(eps)T] [g]. There are 36 pairs, six each equating [S] and [T], and eight each equating [E], [D], and [P]. The 72 relations contain many redundancies. Relations between the elastic, piezoelectric, and dielectric constants are shown schematically in Tables 2 and 3.

CALCULATION SEQUENCE

Using as input [cE], [e], and [(eps)S], one may compute the remaining quantities in a variety of ways. The following sequence is typical:

$$[sE] = [cE]^{(-1)} \tag{44}$$

$$[(bet)S] = [(eps)S]^{(-1)} \tag{45}$$

$$[d] = [e]\ [sE] \tag{46}$$

$$[h] = [(bet)S]\ [e] \tag{47}$$

$$[(eps)T] - [(eps)S] = [e]\ [d]' \tag{48}$$

$$[(eps)T] = [(eps)S] + [e]\ [d]' \tag{49}$$

$$[(bet)T] = [(eps)T]^{(-1)} \tag{50}$$

$$[cD] - [cE] = [e]'\ [h] \tag{51}$$

$$[cD] = [cE] + [e]'\ [h] \tag{52}$$

$$[g] = [(bet)T]\ [d] \tag{53}$$

$$[sE] - [sD] = [d]'\ [g] \tag{54}$$

$$[sD] = [sE] - [d]'\ [g] \tag{55}$$

$$[(betr)S] = [(bet)S] * (eps)o \tag{56}$$

$$[(zetr)S] = [(betr)S]\ [I - (betr)S]^{(-1)} \tag{57}$$

TABLE 2. RELATIONS AMONG MATERIAL CONSTANTS.

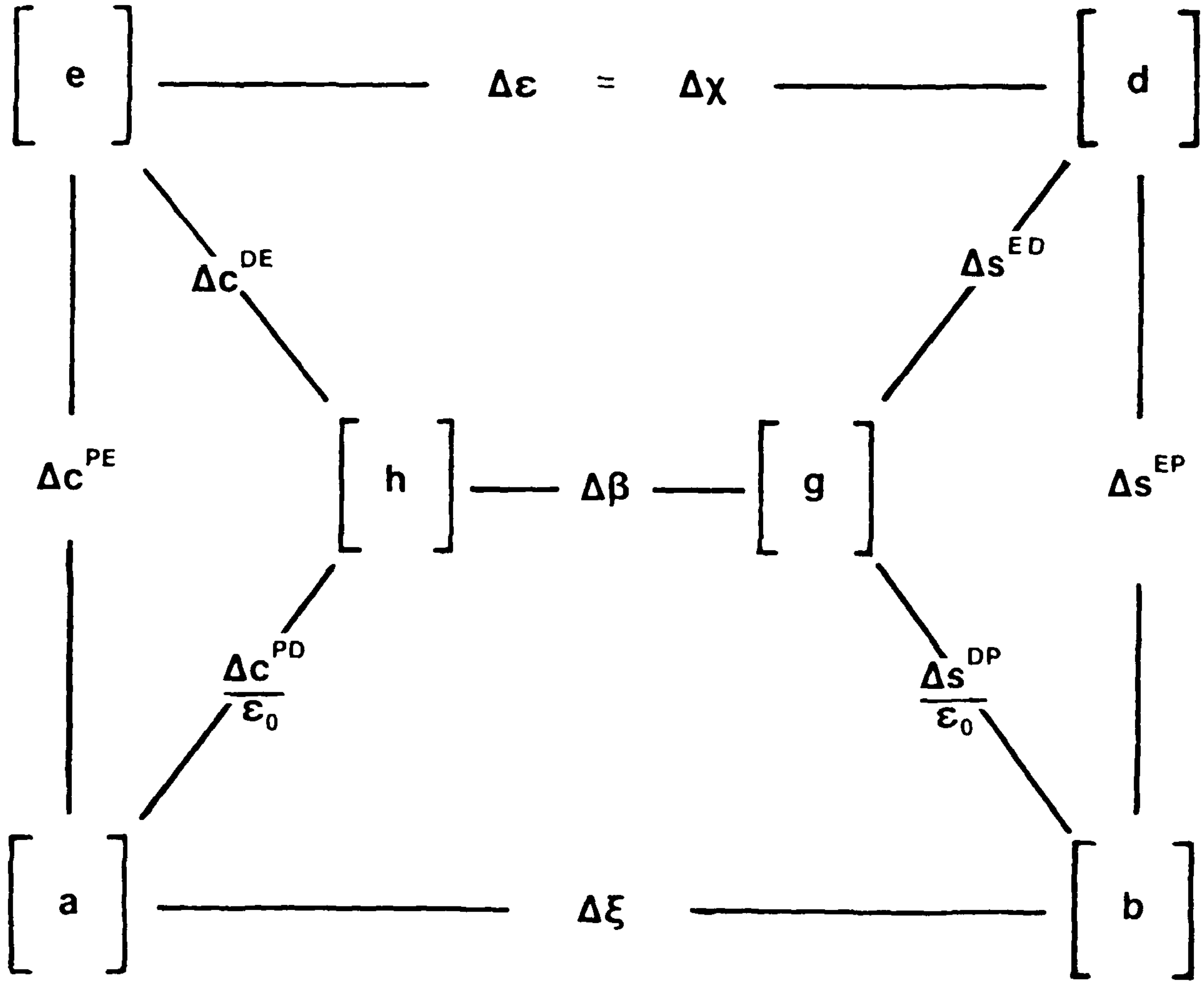
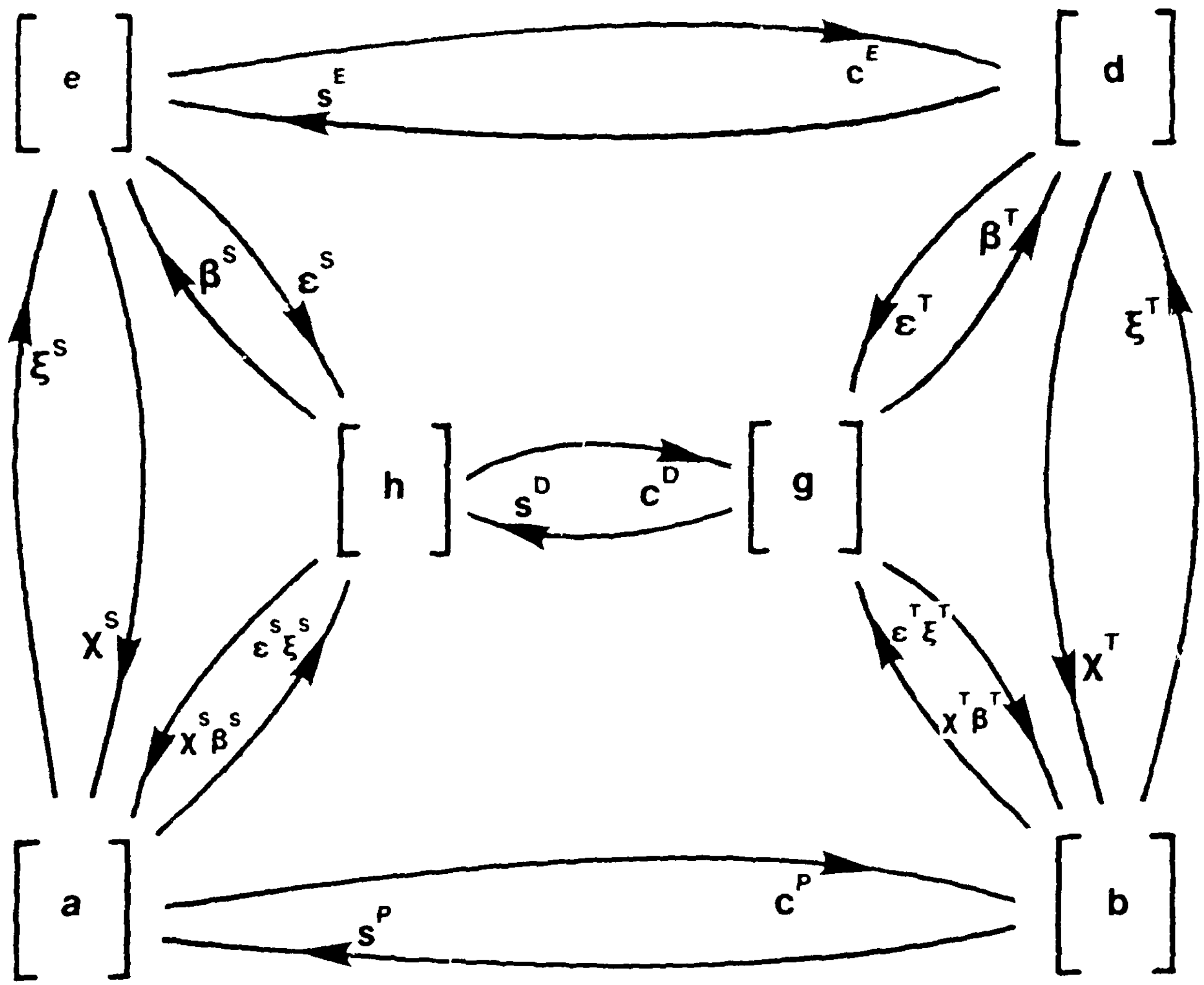


TABLE 3. FURTHER RELATIONS AMONG MATERIAL CONSTANTS.



[(zet)S] = [(zetr)] / (eps)o (58)

[(betr)T] = [(bet)T] * (eps)o (59)

[(zetr)T] = [(betrT] [I - (betr)T] $^{(-1)}$ (60)

[(zet)T = [(zetr)T] / (eps)o (61)

[(chi)S] = [(zet)S] $^{(-1)}$ (62)

[(chi)T] = [(zet)T] $^{(-1)}$ (63)

[a] = [(zet)S] [e] (64)

[b] = [(zet)T] [d] (65)

[cP] - [cE] = [e]' [a] (66)

[cP] = [cE] + [e]' [a] (67)

[cP] - [cD] = [a]' [h] * (eps)o (68)

[sE] - [sP] = [d]' [b] (69)

[sP] = [sE] - [d]' [b] (70)

[sD] - [sP] = [g]' [b] * (eps)o (71)

[(bet)S] - [(bet)T] = [h] [g]' (72)

[(chi)T] - [(chi)S] = [(eps)T] - [(eps)S] (73)

[(zet)S] - [(zet)T] = [a] [b]' (74)

A number of these relations are used as checks. For example, [(bet)S] and [(bet)T] are known from (45) and (50), but the difference is recomputed in (72).

EXPLICIT FORMULAS FOR POINT GROUP 32

Elastic:

The 6x6 elastic constant portions of Tables 4 and 5 partition into 4x4 and 2x2 submatrices. The 4x4 elastic stiffness and compliance submatrices are interrelated by formulas (75) to (93), taken from Cady (Ref. 2):

A = s33 * (s11 + s12) - 2 * s13 * s13 (75)

B = s44 * (s11 - s12) - 2 * s14 * s14 (76)

2 * c11 = s33 / A + s44 / B (77)

2 * c12 = s33 / A - s44 / B (78)

TABLE 4. ELASTOPIEZODIELECTRIC MATRICES FOR POINT GROUP 32: THE [e], [h], AND [a] SETS.

11	12	13	14	00	00	]	11	00	00	cE] e'
12	11	13	-14	00	00	]	-11	00	00	----]------
13	13	33	00	00	00	]	00	00	00	e](eps)S
14	-14	00	44	00	00	]	14	00	00	cD] h'
00	00	00	00	44	14	]	00	-14	00	----]------
00	00	00	00	14	66	]	00	-11	00	h](bet)S
11	-11	00	14	00	00	]	11	00	00	cP] a'
00	00	00	00	-14	-11	]	00	11	00	----]------
00	00	00	00	00	00	]	00	00	33	a](zet)S

66 = (11 - 12) / 2

Matrix entries show only subscripts.

TABLE 5. ELASTOPIEZODIELECTRIC MATRICES FOR POINT GROUP 32: THE [d], [g], AND [b] SETS.

11	12	13	14	00	00	]	11	00	00	sE] d'
12	11	13	-14	00	00	]	-11	00	00	----]------
13	13	33	00	00	00	]	00	00	00	d](eps)T
14	-14	00	44	00	00	]	14	00	00	sD] g'
00	00	00	00	44	14*2	]	00	-14	00	----]------
00	00	00	00	14*2	66	]	00	-11*2	00	g](bet)T
11	-11	00	14	00	00	]	11	00	00	sP] b'
00	00	00	00	-14	-11*2	]	00	11	00	----]------
00	00	00	00	00	00	]	00	00	33	b](zet)T

66 = (11 - 12) * 2

Matrix elements show only subscripts.

$$c13 = - s13 / A \; ; \quad c14 = - s14 / B \qquad (79a), (79b) \tag{79}$$

$$c33 = (s11 + s12) / A \tag{80}$$

$$c44 = (s11 - s12) / B \tag{81}$$

$$c66 = (c11 - c12) / 2 = s44 / (2 * B) \tag{82}$$

$$K = c33 * (c11 + c12) - 2 * c13 * c13 \tag{83}$$

$$L = c44 * (c11 - c12) - 2 * c14 * c14 \tag{84}$$

$$2 * s11 = c33 / K + c44 / L \tag{85}$$

$$2 * s12 = c33 / K - c44 / L \tag{86}$$

$$s13 = - c13 / K \; ; \quad s14 = - c14 / L \qquad (87a), (87b) \tag{87}$$

$$s33 = (c11 + c12) / K \tag{88}$$

$$s44 = (c11 - c12) / L \tag{89}$$

$$s66 = (s11 - s12) = 2 * c44 / L \tag{90}$$

$$\det (4x4) \; [s] = A * B \tag{91}$$

$$\det (4x4) \; [c] = K * L \tag{92}$$

$$A * K = B * L = A * B * K * L = 1 \tag{93}$$

Formulas (75) to (93) hold for each set of constant electrical conditions: either E, D, or P constant.

$$[cD] - [cE] = [\mathrm{del}\ cDE] = [e]' \, [h] = [h]' \, [e] \tag{23}$$

$$\mathrm{del}\ cDE11 = + e11\ h11 \tag{94}$$

$$\mathrm{del}\ cDE12 = - e11\ h11 \tag{95}$$

$$\mathrm{del}\ cDE13 = 0 \tag{96}$$

$$\mathrm{del}\ cDE14 = + e11\ h14 = + h11\ e14 \tag{97}$$

$$\mathrm{del}\ cDE33 = 0 \tag{98}$$

$$\mathrm{del}\ cDE44 = + e14\ h14 \tag{99}$$

$$\mathrm{del}\ cDE66 = + e11\ h11 \tag{100}$$

$$\begin{aligned} [cP] - [cD] = [\mathrm{del}\ cPD] &= [a]' \, [h] * (eps)o \\ &= [h]' \, [a] * (eps)o \end{aligned} \tag{24}$$

$$\mathrm{del}\ cPD11 = (+ a11\ h11) * (eps)o \tag{101}$$

$$\text{del cPD12} = (- a11\ h11) * (eps)o \tag{102}$$

$$\text{del cPD13} = 0 \tag{103}$$

$$\begin{aligned}\text{del cPD14} &= (+ a11\ h14) * (eps)o \\ &= (+ h11\ a14) * (eps)o\end{aligned} \tag{104}$$

$$\text{del cPD33} = 0 \tag{105}$$

$$\text{del cPD44} = (+ a14\ h14) * (eps)o \tag{106}$$

$$\text{del cPD66} = (+ a11\ h11) * (eps)o \tag{107}$$

$$[cP] - [cE] = [\text{del cPE}] = [e]'\ [a] = [a]'\ [e] \tag{25}$$

$$\text{del cPE11} = + e11\ a11 \tag{108}$$

$$\text{del cPE12} = - e11\ a11 \tag{109}$$

$$\text{del cPE13} = 0 \tag{110}$$

$$\text{del cPE14} = + e11\ a14 = + a11\ e14 \tag{111}$$

$$\text{del cPE33} = 0 \tag{112}$$

$$\text{del cPE44} = + e14\ a14 \tag{113}$$

$$\text{del cPE66} = + e11\ a11 \tag{114}$$

From the del c14 entries we have the ratios

$$e11 / e14 = h11 / h14 = a11 / a14. \tag*{(115), (116)}$$

$$[sE] - [sD] = [\text{del sED}] = [d]'\ [g] = [g]'\ [d] \tag{26}$$

$$\text{del sED11} = + d11\ g11 \tag{117}$$

$$\text{del sED12} = - d11\ g11 \tag{118}$$

$$\text{del sED13} = 0 \tag{119}$$

$$\text{del sED14} = + d11\ g14 = + g11\ d14 \tag{120}$$

$$\text{del sED33} = 0 \tag{121}$$

$$\text{del sED44} = + d14\ g14 \tag{122}$$

$$\text{del sED66} = + d11\ g11 * 4 \tag{123}$$

$$[sD] - [sP] = [g]'\ [b] * (eps)o$$

```
              = [b]' [g] * (eps)o                                (27)
  del sDP11 = ( + g11 b11 ) * (eps)o                             (124)
  del sDP12 = ( - g11 b11 ) * (eps)o                             (125)
  del sDP13 = 0                                                  (126)
  del sDP14 = ( + g11 b14 ) * (eps)o
            = ( + b11 g14 ) * (eps)o                             (127)
  del sDP33 = 0                                                  (128)
  del sDP44 = ( + g14 b14 ) * (eps)o                             (129)
  del sDP66 = ( + g11 b11 ) * 4 * (eps)o                         (130)

[sE] - [sP] = [del sEP] = [b]' [d] = [d]' [b]                    (28)
  del sEP11 = + d11 b11                                          (131)
  del sEP12 = - d11 b11                                          (132)
  del sEP13 = 0                                                  (133)
  del sEP14 = + d11 b14 = + b11 d14                              (134)
  del sEP33 = 0                                                  (135)
  del sEP44 = + d14 b14                                          (136)
  del sEP66 = + d11 b11 * 4                                      (137)
```

From the del s14 entries we have the ratios

```
      d11 / d14 = g11 / g14 = b11 / b14.              (138), (139)
```

Piezoelectric:

```
      [d] = [e] [sE]                                             (33)
      d14 = + e14 sE44 + e11 sE14 * 2                            (140)
      d11 = + e11 (sE11 - sE12) + e14 sE14                       (141)

      [h] = [(bet)S] [e]                                         (34)
      h14 = (bet)S11 e14                                         (142)
      h11 = (bet)S11 e11                                         (143)
```

$$[g] = [(bet)T]\ [d] \tag{35}$$

$$g14 = (bet)T11\ d14 \tag{144}$$

$$g11 = (bet)T11\ d11 \tag{145}$$

$$[a] = [(zet)S]\ [e] \tag{36}$$

$$a14 = (zet)S11\ e14 \tag{146}$$

$$a11 = (zet)S11\ e11 \tag{147}$$

$$[b] = [(zet)T]\ [d] \tag{37}$$

$$b14 = (zet)T11\ d14 \tag{148}$$

$$b11 = (zet)T11\ d11 \tag{149}$$

Dielectric:

$$[(bet)Y] = [(eps)Y]^{(-1)} \tag{21}$$

$$(bet)Y11 = 1\ /\ (eps)Y11 \tag{150}$$

$$(bet)Y33 = 1\ /\ (eps)Y33 \tag{151}$$

$$[(zetr)Y] = [(betr)Y]\ [I - (betr)Y]^{(-1)} \tag{152}$$

$$(zet)Y11 = 1\ /\ ((eps)Y11 - (eps)o) \tag{153}$$

$$(zet)Y33 = 1\ /\ ((eps)Y33 - (eps)o) \tag{154}$$

$$[(eps)T - (eps)S] = [del\ (eps)] = [e]\ [d]' = [(chi)T - (chi)S] = [del\ (chi)] = [d]\ [e]' \tag{30}$$

$$del\ (eps)11 = del\ (chi)11 = + e14\ d14 + e11\ d11 * 2 \tag{155}$$

$$del\ (eps)33 = del\ (chi)33 = 0 \tag{156}$$

$$[(bet)S - (bet)T] = [h]\ [g]' = [g]\ [h]' \tag{31}$$

$$del\ (bet)11 = + h14\ g14 + h11\ g11 * 2 \tag{157}$$

$$del\ (bet)33 = 0 \tag{158}$$

$$[(zet)S - (zet)T] = [del\ (zet)] = [a]\ [b]' = [b]\ [a]' \tag{159}$$

$$del\ (zet)11 = + a14\ b14 + a11\ b11 * 2 \tag{160}$$

$$del\ (zet)33 = 0 \tag{161}$$

INPUT VALUES FOR QUARTZ

The values measured by Bechmann (Ref. 1) are as follows:

TABLE 6. ISAGRIC ELASTIC STIFFNESSES.

cE11	cE12	cE13	cE14	cE33	cE44	cE66
86.74	6.98*	11.91	-17.91	107.2	57.94	39.88

Units: $10^{(9)}$ N/m2

* The value of 6.99 appearing in Ref. 1 has been changed so that the relation c66 = (c11 - c12)/2 holds; c11 and c66 are directly measured and hence are more accurately known than c12.

TABLE 7. PIEZOELECTRIC STRESS CONSTANTS.

e11	e14
0.171	-0.0406

Units: C/m2

TABLE 8. DIELECTRIC PERMITTIVITIES AT CONSTANT STRAIN.

(eps)S11	(eps)S33
39.21	41.03

Units: $10^{(-12)}$ F/m

OUTPUT VALUES FOR QUARTZ

The input values from Tables 6, 7, and 8 were used to compute the remaining elastic, piezoelectric, and dielectric quantities for quartz in the manner discussed in prior sections of this report. The results are given in Tables 9 to 16.

TABLE 9. ELASTIC STIFFNESSES.

	cE	cD	cP	del cDE	del cPE	del cPD
11	86.74	87.49	87.70	0.746	0.963	0.218
12	6.98*	6.23	6.02	-0.746	-0.963	-0.218
13	11.91	11.91	11.91	0	0	0
14	-17.91	-18.09	-18.14	-0.177	-0.229	-0.0516
33	107.2	107.2	107.2	0	0	0
44	57.94	57.98	57.99	0.0420	0.0543	0.0123
66	39.88	40.63	40.84	0.746	0.963	0.218

Units: $10^{(9)}$ N/m2

* The value of 6.99 appearing in Ref. 1 has been changed so that the relation c66 = (c11 - c12)/2 holds; c11 and c66 are directly measured and hence are more accurately known than c12.

TABLE 10. ELASTIC COMPLIANCES.

	sE	sD	sP	del sED	del sEP	del sDP
11	12.77	12.64	12.60	0.133	0.171	0.0379
12	-1.79	-1.66	-1.62	-0.133	-0.171	-0.0379
13	-1.22	-1.22	-1.22	0	0	0
14	4.50	4.46	4.45	0.0419	0.0538	0.0119
33	9.60	9.60	9.60	0	0	0
44	20.04	20.03	20.02	0.0132	0.0169	0.00375
66	29.12	28.58	28.43	0.533	0.684	0.152

Units: $10^{(-12)}$ m2/N

TABLE 11. PIEZOELECTRIC [e], [h], AND [a] VALUES.

	e	h	a
11	0.171	4.36	5.63
14	-0.0406	-1.04	-1.34

Units: e: C/m2; h and a: $10^{(9)}$ V/m

TABLE 12. PIEZOELECTRIC [d], [g], AND [b] VALUES.

	d	g	b
11	2.31	57.7	74.1
14	+0.725	18.1	23.3

Units: d: $10^{(-12)}$ m/V; g and b: $10^{(-3)}$ m2/C

TABLE 13. DIELECTRIC (eps) VALUES.

	(eps)S	(eps)T	del (eps)TS
11	39.21	39.97	0.759
33	41.03	41.03	0

Units: $10^{(-12)}$ F/m.

del (eps)TS = del (chi)TS

TABLE 14. DIELECTRIC (chi) VALUES.

	(chi)S	(chi)T	del (chi)TS
11	30.36	31.12	0.759
33	32.18	32.18	0

Units: $10^{(-12)}$ F/m.

del (chi)TS = del (eps)TS

TABLE 15. DIELECTRIC (bet) VALUES.

	(bet)S	(bet)T	del (bet)TS
11	25.50	25.02	-0.485
33	24.37	24.37	0

Units: $10^{(9)}$ m/F.

TABLE 16. DIELECTRIC (zet) VALUES.

	(zet)S	(zet)T	del (zet)TS
11	32.94	32.14	-0.804
33	31.08	31.08	0

Units: $10^{(9)}$ m/F.

CONCLUSIONS

This report provides formulas interrelating the coefficients that appear in the several alternative sets of constitutive equations involving the elastic, piezoelectric, and dielectric properties of crystals. These are then specialized for crystals of class 32; using measured values reported for quartz, numerical values of the elements of the polarization matrices are calculated.

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